

Complex Korteweg-de Vries equation and Nonlinear dust-acoustic waves in a magnetoplasma with a pair of trapped ions

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The nonlinear propagation of dust-acoustic (DA) waves in a magnetized dusty plasma with a pair of trapped ions is investigated. Starting from a set of hydrodynamic equations for massive dust fluids as well as kinetic Vlasov equations for ions, and applying the reductive perturbation technique, a Korteweg-de Vries (KdV)-like equation with a complex coefficient of nonlinearity is derived, which governs the evolution of small-amplitude DA waves in plasmas. The complex coefficient arises due to vortex-like distributions of both positive and negative ions. An analytical as well as numerical solution of the KdV equation are obtained and analyzed with the effects of external magnetic field, the dust pressure as well as different mass and temperatures of positive and negative ions.

I. INTRODUCTION

Recently, there has been a renewed interest in investigating electrostatic disturbances in pair-plasmas and, in particular, plasmas with a pair of ions [1–7]. However, nonthermal pair plasmas may frequently occur not only in semiconductors in the form of electron and ion holes, but also in many astrophysical environments, e.g., pulsars, magnetars, as well as in the early universe, active galactic nuclei and supernova remnants in the form of electrons and positrons [8–11]. On the other hand, a number of experiments have been conducted to create pair-ion plasmas using fullerene as ion source [12]. Furthermore, it has been observed that the dust particles injected into a pair-ion plasma (e.g., K^+/SF_6^- plasmas) can become positively charged when the number density of negative ions greatly exceeds that of electrons ($n_{n0} \gtrsim 500n_{e0}$) [13, 14]. These pose some possibilities to investigate collective behaviors as well as the formation of nonlinear coherent structures in pair-ion plasmas under controlled conditions. The formation of phase space holes in pure pair-ion plasmas [5] as well as ion holes in dusty pair-ion plasmas [6] in the propagation of large amplitude electrostatic waves have been investigated in the recent past in which ions have been treated as trapped in self-created localized electrostatic potentials as prescribed by Schamel [15].

In this paper we present a theoretical study on the formation and the dynamics of small-amplitude solitary structures in a dusty plasma composed of charged dust particles and a pair of ions without electrons. In our theoretical model the massive charged dusts are described by a set of fluid equations, while the dynamics of both positive and negative ions are governed by kinetic Vlasov equations. Using the reductive perturbation technique we show that the evolution of small-amplitude electrostatic waves can be described by a Korteweg-de Vries (KdV)-like equation with a complex coefficient of the

nonlinearity. A stationary as well as numerical solutions of the KdV equation are obtained and analyzed with the effects of external magnetic field, the dust pressure as well as different mass and temperatures of ions.

II. BASIC EQUATIONS

We consider the nonlinear propagation of dust-acoustic (DA) solitary waves in a magnetized dusty plasma which consists of positively or negatively charged mobile dusts and a pair of trapped ions with vortex-like distributions. The dust particles are assumed to have equal mass and constant charge. The collisions of all particles are also neglected compared to the dust plasma period. Furthermore, in dusty pair-ion plasmas the ratio of electric charge to mass of dust particles remains much smaller than those of positive and negative ions. We also assume that the size of the dust grains is small compared to the average interparticle distance. The static magnetic field is considered along the z -axis, i.e., $\mathbf{B} = B_0 \hat{z}$. While the dynamics of massive charged dusts in the propagation of DA waves ($v_{td} \ll v_p \ll v_{p,n}$, where $v_{tj} (= \sqrt{k_B T_j / m_j})$ is the thermal velocity of j -th species particles and v_p is the phase velocity of the wave) is described by a set of fluid equations (1) and (2), the dynamics of singly charged positive and negative ions are described by the Vlasov equations (3).

$$\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \mathbf{v}_d) = 0, \quad (1)$$

$$\frac{\partial \mathbf{v}_d}{\partial t} + (\mathbf{v}_d \cdot \nabla) \mathbf{v}_d = \frac{q_d}{m_d} (\mathbf{E} + \mathbf{v}_d \times \mathbf{B}_0) - \frac{\nabla P}{m_d n_d}, \quad (2)$$

$$\frac{\partial f_j}{\partial t} + \mathbf{v} \cdot \nabla f_j - \frac{q_j}{m_j} \nabla \phi \cdot \frac{\partial f_j}{\partial \mathbf{v}}. \quad (3)$$

The system of equations is then closed by the Poisson equation

$$\nabla \cdot \mathbf{E} = 4\pi e (n_p - n_n + \alpha Z_d n_d). \quad (4)$$

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In Eqs. (1)-(4), q_j , n_j , \mathbf{v}_j , f_j and m_j respectively, denote the charge, number density (with its equilibrium value n_{j0}), velocity, velocity distribution function, and mass of j -species particles. Also, $q_d = \alpha z_d e$ with $\alpha = \pm$ denoting for positively/negatively charged dusts and z_d the charge state. Also, $\mathbf{E} = -\nabla\phi$ is the electric field with ϕ denoting the electrostatic potential and P is the dust thermal pressure given by the adiabatic equation of state $P/P_0 = (n_d/n_{d0})^\gamma$. Here, $\gamma = 5/3$ is the adiabatic index for three-dimensional configuration and $P_0 = n_{d0} k_B T_d$ is the equilibrium dust pressure with k_B denoting the Boltzmann constant and T_j the thermodynamic temperature of j -species particles. Furthermore, the ion densities are given by

$$n_j = \int_{-\infty}^{\infty} f_j d\mathbf{v}. \quad (5)$$

In what follows, we recast Eqs. (1)-(4) in terms of dimensionless variables. To this end the physical quantities are normalized according to $n_j \rightarrow n_j/n_{j0}$, $(\mathbf{v}, \mathbf{v}_d) \rightarrow (\mathbf{v}, \mathbf{v}_d)/c_d$, $\phi \rightarrow e\phi/k_B T_p$ with e denoting the elementary charge, $f_j \rightarrow f_j v_{tp}/n_{j0}$, where $c_d = \sqrt{z_d k_B T_p/m_d} = \omega_{pd} \lambda_D$ is the dust-acoustic speed with $\omega_{pd} = \sqrt{4\pi n_{d0} z_d^2 e^2/m_d}$ and $\lambda_D = \sqrt{k_B T_p/4\pi n_{d0} z_d e^2}$ denoting, respectively, the dust plasma frequency and the plasma Debye length. The space and time variables are normalized by λ_D and ω_{pd}^{-1} respectively. Thus, from Eqs. (1)-(5) we have following set of normalized equations.

$$\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \mathbf{v}_d) = 0, \quad (6)$$

$$\frac{d\mathbf{v}_d}{dt} + \alpha \nabla \phi = \alpha \omega_c \mathbf{v}_d \times \hat{z} - \frac{5}{3} \sigma_d n_d^{-1/3} \nabla n_d, \quad (7)$$

$$\delta \frac{\partial f_j}{\partial t} + \mathbf{v} \cdot \nabla f_j - \zeta_j \frac{m_p}{m_j} \nabla \phi \cdot \frac{\partial f_j}{\partial \mathbf{v}} = 0, \quad (8)$$

$$\nabla^2 \phi = \mu_n n_n - \mu_p n_p - \alpha n_d, \quad (9)$$

$$n_j = \int_{-\infty}^{\infty} f_j d\mathbf{v}, \quad (10)$$

where $d/dt \equiv \partial/\partial t + \mathbf{v}_d \cdot \nabla$, $\alpha = \pm 1$ for positively/negatively charged dusts, $\omega_c = |q_d| B_0/m_d \omega_{pd}$ is the dust-cyclotron frequency normalized by the dust plasma frequency, $\sigma_d \equiv T_d/T_p z_d$, $\delta = \sqrt{z_d m_p/m_d}$, $\zeta_j = \pm 1$ for positive/negative ions and $\mu_j = n_{j0}/Z_d n_0$ are the density ratios ($j = p, n$) which satisfy the following charge neutrality condition at equilibrium:

$$\mu_p + \alpha = \mu_n. \quad (11)$$

We neglect the ion inertial effects compared to the charged dusts, i.e., $\delta \rightarrow 0$ in Eq. (8). The distribution functions f_j for positive and negative ions, which

are constant of motion of the Vlasov Eq. (8), are chosen [15] for the excitation of localized solitary waves so that (i) they are continuous, and both the free particle distributions are Maxwellian distribution where $\phi \rightarrow 0$ at $|\xi| \rightarrow \pm\infty$ and trapped particles are absent, (ii) both trapped particle distributions are Maxwellian (with also negative temperatures). Thus, f_j (for free and trapped particles) are (with a suitable choice of the normalization constants) [15–18] for positive ions

$$f_{pf}(v) = \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{1}{2} (v^2 + 2\phi) \right], \quad |v| > \sqrt{-2\phi}, \quad (12)$$

$$f_{pt}(v) = \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{\sigma_p}{2} (v^2 + 2\phi) \right], \quad |v| \leq \sqrt{-2\phi}, \quad (13)$$

and for negative ions

$$f_{nf}(v) = \sqrt{\frac{m\sigma}{2\pi}} \exp \left[-\frac{m\sigma}{2} \left(v^2 - \frac{2\phi}{m} \right) \right], \quad |v| > \sqrt{2\phi/m}, \quad (14)$$

$$f_{nt}(v) = \sqrt{\frac{m\sigma}{2\pi}} \exp \left[-\frac{1}{2} m\sigma \left(v^2 - \frac{2\phi}{m} \right) \right], \quad |v| \leq \sqrt{2\phi/m}, \quad (15)$$

where $m (= m_n/m_p \gtrsim 1)$ is the mass ratio, $\sigma (= T_p/T_n \gtrsim 1)$ is the temperature ratio and σ_j , for $j = p, n$, measure the inverse of the trapped positive and negative ion temperatures which may be negative ($\sigma_j < 0$) corresponding to a depression in the trapped particle distribution. The case of $\sigma_j \rightarrow 0$ represents the plateau (constant or flat-topped) and $\sigma_j \rightarrow 1$ corresponds to the Boltzmann distribution of ions. Next, integrating the particle distribution functions (12)-(15) over the velocity space, i.e., using Eq. (10) we obtain the number densities n_j for positive and negative ions as

$$n_p(\phi) = I(-\phi) + \frac{1}{\sqrt{|\sigma_p|}} \begin{cases} e^{-\sigma_p \phi} \operatorname{erf}(\sqrt{-\sigma_p \phi}), & \sigma_p \geq 0 \\ \frac{2}{\sqrt{\pi}} W(\sqrt{\sigma_p \phi}), & \sigma_p < 0, \end{cases} \quad (16)$$

$$n_n(\phi) = I(\sigma \phi) + \frac{1}{\sqrt{|\sigma_n|}} \begin{cases} e^{\sigma \sigma_n \phi} \operatorname{erf}(\sqrt{\sigma \sigma_n \phi}), & \sigma_n \geq 0 \\ \frac{2}{\sqrt{\pi}} W(\sqrt{-\sigma \sigma_n \phi}), & \sigma_n < 0, \end{cases} \quad (17)$$

where $I(x) = \exp(x) [1 - \operatorname{erf}(\sqrt{x})]$. The error and Dawson functions $\operatorname{erf}(x)$ and $W(x)$ are, respectively, given by

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt, \quad W(x) = e^{-x^2} \int_0^x e^{t^2} dt. \quad (18)$$

In the small amplitude limit $\phi \ll 1$, so that $\sigma \phi \ll 1$, we obtain from Eqs. (16) and (17) the following expressions for the number densities [15–18]

$$n_p \approx 1 - \phi - \frac{4(1 - \sigma_p)}{3\sqrt{\pi}} (-\phi)^{3/2} + \frac{1}{2} \phi^2, \quad (19)$$

$$n_n \approx 1 + (\sigma \phi) - \frac{4(1 - \sigma_n)}{3\sqrt{\pi}} (\sigma \phi)^{3/2} + \frac{1}{2} (\sigma \phi)^2. \quad (20)$$

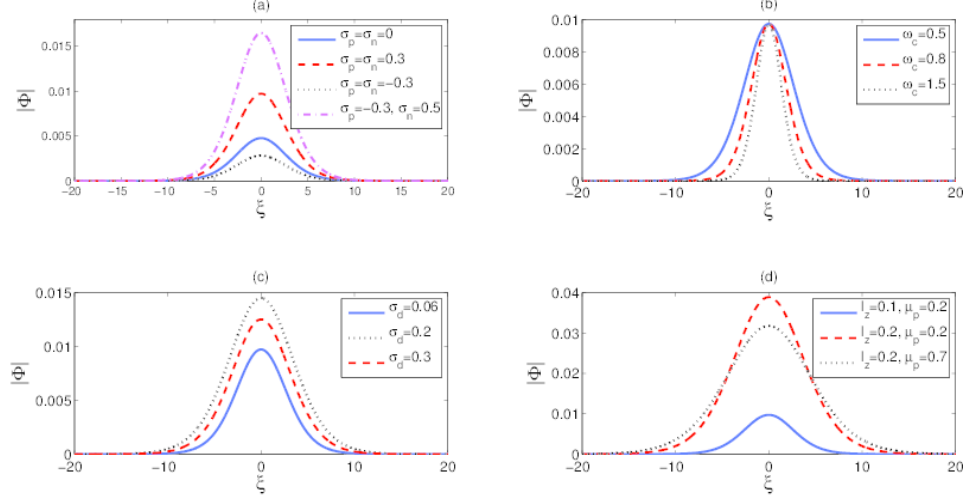


FIG. 1. Profiles of $|\Phi|$ given by Eq. (33) are shown with respect to ξ (and with $\sigma \sim 1$) for different values of the plasma parameters as in the figure. The fixed parameter values for the subplots (a) to (d), respectively, are (a) $\alpha = 1$, $\mu_p = 0.2$, $l_z = 0.1$, $\omega_c = 0.5$, $\sigma_d = 0.06$ and $u_0 = 0.1$, (b) $\alpha = 1$, $\mu_p = 0.2$, $l_z = 0.1$, $\sigma_p = \sigma_n = 0.3$, $\sigma_d = 0.06$ and $u_0 = 0.1$, (c) $\alpha = 1$, $\mu_p = 0.2$, $l_z = 0.1$, $\sigma_p = \sigma_n = 0.3$, $\omega_c = 0.5$, $\sigma_d = 0.06$ and $u_0 = 0.1$, and (d) $\alpha = 1$, $\mu_p = 0.2$, $\sigma_p = \sigma_n = 0.3$, $\omega_c = 0.5$, $\sigma_d = 0.06$ and $u_0 = 0.1$.

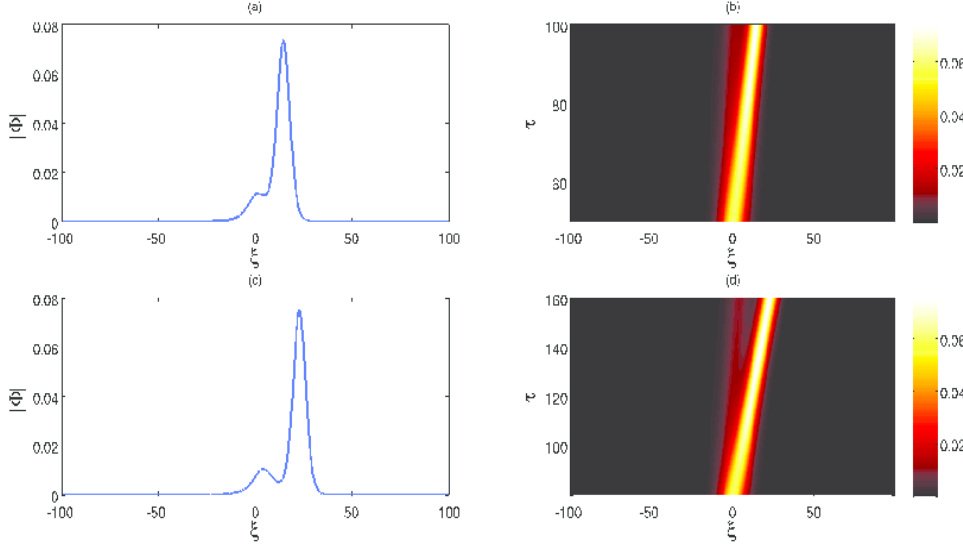


FIG. 2. The space-time evolution of the soliton profile $|\Phi|$ [numerical solution of Eq. (30)] is shown at $\tau = 100$ [Subplots (a) and (b)] and $\tau = 160$ [Subplots (c) and (d)]. While the plots (a) and (c) show the soliton profiles with space, plots (b) and (d) are the corresponding contour plots. The parameter values are the same as for the dashed line in Fig. 1(d) and $\sigma \sim 1$.

III. EVOLUTION EQUATION

In order to derive the evolution equation for the DA waves, we transform the space and time variables according to [16]

$$\xi = \epsilon^{1/4} (l_x x + l_y y + l_z z - Mt), \quad \tau = \epsilon^{3/4} t, \quad (21)$$

where ϵ is a small parameter measuring the strength of nonlinearity. The dependent variables are expanded as

[16]

$$\begin{aligned}
n &= 1 + \epsilon n^{(1)} + \epsilon^{3/2} n^{(2)} + \dots, \\
\phi &= \epsilon \phi^{(1)} + \epsilon^{3/2} \phi^{(2)} + \dots, \\
v_z &= 1 + \epsilon v_z^{(1)} + \epsilon^{3/2} v_z^{(2)} + \dots, \\
v_{x,y} &= \epsilon^{5/4} v_{x,y}^{(1)} + \epsilon^{3/2} v_{x,y}^{(2)} + \dots.
\end{aligned} \tag{22}$$

The anisotropy in Eq. (22) for the transverse velocity components of dust fluids is introduced on the assumption that the dust gyromotion is a higher-order effect than the motion parallel to the magnetic field. Next, we substitute Eqs. (21) and (22) into Eqs. (6)-(9) and equate different powers of ϵ successively. In the lowest order ($\epsilon^{5/4}$), we obtain the following first-order quantities

$$n^{(1)} = \alpha \frac{l_z}{M} v_z^{(1)} = (\mu_p + \sigma \mu_n) \phi^{(1)}, \tag{23}$$

$$v_{x,y}^{(1)} = \mp \frac{l_{y,x}}{\omega_c} \left(\frac{\partial \phi^{(1)}}{\partial \xi} + \frac{5}{3} \alpha \sigma_d \frac{\partial n^{(1)}}{\partial \xi} \right), \tag{24}$$

and the dispersion relation for the nonlinear wave speed given by

$$M = l_z \left(\frac{5}{3} \sigma_d + \frac{1}{\mu_p + \sigma \mu_n} \right)^{1/2}. \tag{25}$$

Replacing M by ω/k , one can obtain the same dispersion relation after Fourier analyzing the linearized basic equations (6)-(9), i.e., assuming the perturbations as oscillations with the wave frequency ω and the wave number k . We find that the phase speed M (normalized by the DIA speed c_d) can be larger or smaller than the unity depending on the choice of the parameter values. The value of M increases with increasing values of both l_z and σ_d . However, its values can slowly decrease with increasing values of the density ratios μ_j as well as the temperature ratio σ . From Eq. (6), collecting the coefficients of $\epsilon^{7/4}$ we obtain

$$M \frac{\partial n^{(2)}}{\partial \xi} = \frac{\partial n^{(1)}}{\partial \tau} + \sum_{j=x,y,z} l_j \frac{\partial v_j^{(2)}}{\partial \xi}. \tag{26}$$

Similarly, equating the coefficients of $\epsilon^{3/2}$ from the x - and y -components of Eq. (7), and the coefficients of $\epsilon^{7/4}$ from the z -component of Eq. (7) we successively obtain

$$\alpha v_{x,y}^{(2)} = \pm \frac{M}{\omega_c} \frac{\partial v_{y,x}^{(1)}}{\partial \xi} = \left[\frac{M l_{x,y}}{\omega_c^2} + \frac{5}{3} \sigma_d (\mu_p + \sigma \mu_n) \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} \right], \tag{27}$$

$$M \frac{\partial v_z^{(2)}}{\partial \xi} = \frac{\partial v_z^{(1)}}{\partial \tau} + l_z \left(\alpha \frac{\partial \phi^{(2)}}{\partial \xi} + \frac{5}{3} \sigma_d \frac{\partial n^{(2)}}{\partial \xi} \right). \tag{28}$$

From the coefficients of $\epsilon^{3/2}$ of Eq. (9), we obtain an equation in which $n^{(2)}$ is eliminated by the use of Eqs.

(26), (27) and (28), and the coefficient of $\phi^{(2)}$ vanishes by Eq. (25). Thus, arranging the terms and using Eq. (23) one obtains the following KdV-like equation

$$\frac{\partial \Phi}{\partial \tau} + \left(A_p \sqrt{-\Phi} + A_n \sqrt{\Phi} \right) \frac{\partial \Phi}{\partial \xi} + B \frac{\partial^3 \Phi}{\partial \xi^3} = 0, \tag{29}$$

where $\Phi \equiv \phi^{(1)}$. It follows that Eq. (29) has a complex solution for Φ . Typically, for $\Phi \sim r \exp(i\theta)$, where r is a real function of ξ and τ , and θ is a constant, one can have $\sqrt{-\Phi} = i\sqrt{\Phi}$. Thus, Eq. (29) can be written as

$$\frac{\partial \Phi}{\partial \tau} + A \sqrt{\Phi} \frac{\partial \Phi}{\partial \xi} + B \frac{\partial^3 \Phi}{\partial \xi^3} = 0, \tag{30}$$

where the coefficients of nonlinearity ($A \equiv A_n + iA_p$) and dispersion (B) are given by

$$A_j = \frac{\alpha}{\sqrt{\pi}} \frac{(1 - \sigma_j) \mu_j}{M(\mu_p + \sigma \mu_n)} \left(\frac{T_p}{T_j} \right)^{3/2}, \tag{31}$$

$$B = \frac{l_z^2}{2M} \left[1 + \frac{M^4 (1 - l_z^2)}{\omega_c^2 l_z^4} (\mu_p + \sigma \mu_n)^2 \right]. \tag{32}$$

The nonlinear coefficient A becomes complex due to vortex-like distributions of two oppositely charged particles. In absence of one of them, A becomes real, and one can then obtain solitary waves with positive or negative potential. A stationary soliton solution of Eq. (30) can easily be obtained with its absolute value as (For details see Appendix A)

$$|\Phi| = \Phi_0 \operatorname{sech}^4 [(\xi - u_0 \tau) / W], \tag{33}$$

where u_0 is a constant, and $\Phi_0 = (15u_0/8|A|)^2$ and $W = \sqrt{16B/u_0}$ are the amplitude and width of the soliton respectively.

IV. RESULTS AND DISCUSSION

We numerically analyze the solution (33) with different plasma parameters as shown in Fig. 1. Since σ_j ($j = p, n$) represents the reciprocal temperature of the trapped positive and negative ions, and can be allowed from their negative to positive values corresponding to different trapped particle distributions, we consider negative, zero as well as positive values of σ_j .

From Fig. 1(a), it is seen that as σ_j increases from $\sigma_j = 0$ (corresponding to a constant or flat-topped distribution of ions) to $\sigma_j \sim 1$ (corresponding to the Boltzmann distributions of ions), both the amplitude and width of the soliton increase (See the solid and dashed lines). Note here that the values of $\sigma_j > 1$, for which the influence of the trapped ions are inverted, may be physically unrealistic as those correspond to a more steepened wave which can become unstable due to more peaked bump of the ion distributions. However, as the absolute

value of A starts increasing for $\sigma_j < 0$, which corresponds to a depression in the trapped particle distribution, both the amplitude and width of the soliton are reduced (See the dotted line). The same can further be enhanced for values of σ_j satisfying $\sigma_p \sigma_n < 0$ (See the dash-dotted line).

Figure 1(b) shows the soliton profile with the influence of the external magnetic field. Since ω_c contributes only to the dispersive coefficient B of Eq. (30), the effect of the magnetic field with increasing its intensity is to reduce the width (without changing the amplitude) of the soliton. Thus, the external magnetic field makes the solitary structure more spiky. However, for stronger magnetic fields with $\omega_c \gg 1$, the width remains almost unaltered as in this case $B \sim l_z^2/2M$.

The thermal effects of charged dusts are shown in Fig. 1(c). It is found that the effect of the dust thermal pressure σ_d is to enhance both the amplitude and width of the soliton. The enhancement is due to the fact that as σ_d increases, the values of $|A|$ (B) decrease (increase), and hence the increase in both the amplitude and width. However, an opposite trend occurs by the effects of the positive to negative ion temperature ratio σ (not shown in the figure). Typically, it reduces both the soliton amplitude and width significantly with a small increment of its value.

Figure 1(d) exhibits the effects of the obliqueness of propagation l_z and the relative (to dusts) concentration of positive ions μ_p . We find that both the amplitude and width of the soliton are greatly enhanced by a small increment of l_z [Since A_j (B) is inversely (directly) proportional to l_z]. However, as the positive ion concentration increases, the amplitude gets reduced but the width increases.

Next, we numerically solve Eq. (30) by the Runge-Kutta scheme with an initial condition of the form $\Phi(\xi) = 0.05 \text{sech}^4(\xi/10) \exp(-i\xi/15)$ and time step $d\tau = 0.001$. The development of the wave form $|\Phi|$ after a finite interval of time is shown in Fig. 2. The parameter values are considered as the same as for the dashed line in Fig. 1(d). It is seen that the leading part of the initial wave steepens due to positive nonlinearity. As the time goes on the pulse separates into solitons and a residue due to the wave dispersion [See the subplots (a) and (b)]. It is found that once the solitons are formed and separated, they propagate in the forward direction without changing their shape due to the nice balance of the nonlinearity and dispersion [See the subplots (c) and (d)].

V. CONCLUSION

We have investigated the nonlinear propagation of dust-acoustic waves in a magnetized plasma which consists of warm positively charged dusts and a pair of free, as well as, trapped ions. We have shown that the evolution of small-amplitude DA waves can be described by a KdV-type equation with a complex coefficient of the

nonlinearity. Such complex coefficient appears due to vortex-like distributions of both the ion species. The KdV equation is solved both analytically and numerically. The properties of the absolute value of Φ are only exhibited graphically. It is shown that while the external magnetic field only influences the width of the soliton, the trapped ion temperatures, the thermal pressures of ions and dusts, the relative concentration of positive ions as well as the obliqueness of propagation have significant effects on both the amplitude and width of the solitons. We stress that other solutions [19–22] than those presented here of the complex KdV equation could of interest but beyond the scope of the present work. To conclude, the present results should be useful in understanding the nonlinear features of electrostatic localized disturbances in laboratory and space plasmas.

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Appendix A: Stationary solution of the KdV-like equation

Equation (30) is recast as

$$\frac{\partial \Phi}{\partial \tau} + A\sqrt{\Phi} \frac{\partial \Phi}{\partial \xi} + B \frac{\partial^3 \Phi}{\partial \xi^3} = 0. \quad (\text{A1})$$

Next, we apply the transformation $\eta = \xi - u_0\tau$ to obtain from Eq. (A1)

$$\frac{d}{d\eta} \left(B\ddot{\Phi} - u_0\Phi + \frac{2}{3}A\Phi^{3/2} \right) = 0, \quad (\text{A2})$$

where the dot denotes differentiation with respect to η . Integrating Eq. (A2) with respect to η and using the boundary conditions $\Phi, \ddot{\Phi} \rightarrow 0$ as $\xi \rightarrow \pm\infty$ we get

$$B\ddot{\Phi} - u_0\Phi + \frac{2}{3}A\Phi^{3/2} = 0. \quad (\text{A3})$$

Multiplying Eq. (A3) by $2\dot{\Phi}$ and integrating once with respect to η , we obtain

$$B\dot{\Phi}^2 - u_0\Phi^2 + \frac{8}{15}A\Phi^{5/2} = 0, \quad (\text{A4})$$

where we have used the boundary conditions $\Phi, \dot{\Phi} \rightarrow 0$. From Eq. (A4) we have

$$\dot{\Phi} = \pm \Phi \sqrt{\frac{u_0}{B} - \frac{8A}{15B}\sqrt{\Phi}}, \quad (\text{A5})$$

$$\text{or, } \int \frac{d\Phi}{\Phi \sqrt{u_0/B - (8A/15B) \sqrt{\Phi}}} = \pm \int d\eta, \quad (\text{A6})$$

which gives ($a = u_0/B$ and $b = 8A/15B$)

$$-\frac{4}{\sqrt{a}} \tanh^{-1} \sqrt{\frac{a - b\sqrt{\Phi}}{a}} = \pm \eta, \quad (\text{A7})$$

$$\text{or, } \sqrt{\frac{a - b\sqrt{\Phi}}{a}} = \mp \tanh \left(\frac{\sqrt{a}}{4} \eta \right). \quad (\text{A8})$$

Thus, we obtain a soliton solution of Eq. (30) as

$$1 - \tanh^2 \left(\frac{\sqrt{a}}{4} \eta \right) = \frac{b}{a} \sqrt{\Phi}, \quad (\text{A9})$$

$$\text{or, } \Phi = \left(\frac{15u_0}{8A} \right)^2 \text{sech}^4 \left(\sqrt{\frac{u_0}{16B}} \eta \right). \quad (\text{A10})$$

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